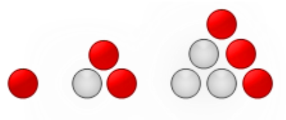
1. What are the next five numbers in this sequence of triangular numbers?
   1, 3, 6, 10, 15…

2. Write down the term to term rule that describes the triangular number sequence.

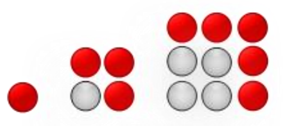
3. Draw the next two diagrams in this sequence:



1 3 6

4. The position to term rule for generating triangular numbers is $T(n) = \frac{1}{2}(n^2 + n)$.
   Use this formula to find the 10th, 15th and 100th triangular numbers.

5. What formula will generate this sequence of square numbers?

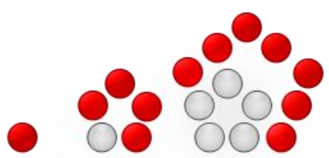


1 4 9

6. Describe how you can use the sequence of triangular numbers 1, 3, 6, 10, 15, etc. to create the sequence of square numbers 1, 4, 9, 16, 25, etc.

7. The formula for pentagonal numbers is $T(n) = \frac{1}{2}(3n^2 - n)$. Use this to work out the first six pentagonal numbers.

8. Draw the next two diagrams in this sequence of pentagonal numbers:



9. These diagrams represent the first three terms of the hexagonal numbers. Use them to find the first five hexagonal numbers.

10. Use the patterns in the sequence of formulae for triangular numbers, $T(n) = \frac{1}{2}(n^2 + n)$, square numbers, $T(n) = n^2$, and pentagonal numbers, $T(n) = \frac{1}{2}(3n^2 - n)$ to find the formulae for hexagonal numbers, heptagonal numbers and octagonal numbers.

Extension

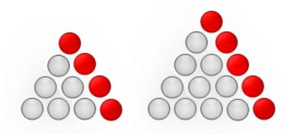
In 1638, Pierre de Fermat stated that every positive integer can be written as the sum of a maximum of:

- three triangular numbers, e.g. 17 = 1 + 6 + 10
- four square numbers, e.g. 17 = 1 + 16
- five pentagonal numbers, e.g. 17 = 5 + 12
- six hexagonal numbers, e.g. 17 = 1 + 1 + 15
- seven heptagonal numbers etc.

1. Find the sums required to make **47** from only triangular numbers, only square numbers etc. up to and including only decagonal numbers.
2. Find a number that can be made from exactly three triangular numbers, one that can be made from four square numbers, one that can be made from five pentagonal numbers etc. up to and including decagonal numbers.

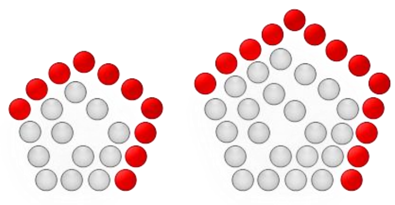
Solutions and teacher notes

1. 21, 28, 36, 45, 55
2. Starting from 1, add 2, add 3, add 4, etc
3.



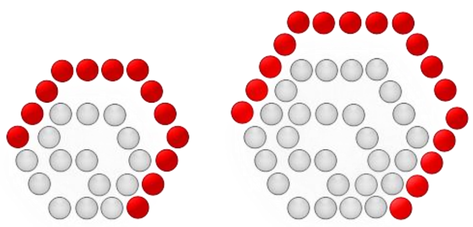
10 **15**

4. 55, 120, 5 050
5. n^2
6. 1 + 3 = 4, 3 + 6 = 9, 6 + 10 = 16, etc.
7. 1, 5, 12, 22, 35, 51
8.



22 **35**

9. 1, 6, 15, 28, 45



28 **45**

10. Students may need a hint here. Try writing the formulae like this:

    $\mathrm{T(n)} = ½(1\mathrm{n}^2 + 1\mathrm{n}), \mathrm{T(n)} = ½(2\mathrm{n}^2 + 0\mathrm{n}), \mathrm{T(n)} = ½(3\mathrm{n}^2 - 1\mathrm{n})$

    Answers: $\mathrm{T(n)} = ½(4\mathrm{n}^2 - 2\mathrm{n}), \mathrm{T(n)} = ½(5\mathrm{n}^2 - 3\mathrm{n}), \mathrm{T(n)} = ½(6\mathrm{n}^2 -$

Extension

Ways to make 47:

Type of numbers used	**Totals**
Triangular	1 + 1 + 45
Square	1 + 1 + 9 + 36
Pentagonal	12 + 35
Hexagonal	1 + 1 + 45
Heptagonal	1 + 7 + 7 + 7 + 7 + 18
Octagonal	1 + 1 + 1 + 1 + 1 + 21 + 21
Nonagonal	1 + 46
Decagonal	10 + 10 + 27

